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Research Article

A Bayesian Game-Theoretic Approach for Distributed Resource Allocation in Fading Multiple Access Channels

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A Bayesian game-theoretic model is developed to design and analyze the resource allocation problem in K-user fading multiple access channels (MACs), where the users are assumed to selfishly maximize their average achievable rates with incomplete information about the fading channel gains. In such a game-theoretic study, the central question is whether a Bayesian equilibrium exists, and if so, whether the network operates efficiently at the equilibrium point. We prove that there exists exactly one Bayesian equilibrium in our game. Furthermore, we study the network sum-rate maximization problem by assuming that the users coordinate according to a symmetric strategy profile. This result also serves as an upper bound for the Bayesian equilibrium. Finally, simulation results are provided to show the network efficiency at the unique Bayesian equilibrium and to compare it with other strategies.

1. Introduction

The fading multiple access channel (MAC) is a basic wireless channel model that allows several transmitters connected to the same receiver to transmit over it and share its capacity. The capacity region of the fading MAC and the optimal resource allocation algorithms have been characterized and well studied in many pioneering works with different information assumptions [1–4]. However, in order to achieve the full capacity region, it usually requires a central computing resource (a scheduler with comprehensive knowledge of the network information) to globally allocate the system resources. This process is centralized, since it involves feedback and overhead communication whose load scales linearly with the number of transmitters in the network. In addition, with the fast evolution of wireless techniques, this centralized network infrastructure begins to expose its weakness in many aspects, for example, slow reconfiguration against varying environment, increased computational complexity, and so forth. This is especially crucial for femtocell networks where it is quite difficult to centralize the information due to a limited capacity backhaul. Moreover, the high density of base stations would increase the cost of centralizing the information.

In recent years, increased research interest has been given to self-organizing wireless networks in which mobile devices allocate resources in a decentralized manner [5]. Tools from game theory [6] have been widely applied to study the resource allocation and power control problems in fading MAC [7], as well as many other types of channels, such as orthogonal frequency division multiplexing (OFDM) [8], multiple input and multiple output (MIMO) channels [9, 10], and interference channels [11]. Typically, the game-theoretic models used in these previous works assume that the knowledge, for example, channel state information (CSI), about other devices is available to all devices. However, this assumption is hardly met in practice. In practical wireless scenarios, mobile devices can have local information but can barely access to global information on the network status.

A static noncooperative game has been introduced in the context of the two-user fading MAC, known as "waterfilling game" [7]. By assuming that users compete with transmission rates as utility and transmit powers as moves, the authors show that there exists a unique Nash equilibrium

[12] which corresponds to the maximum sum-rate point of the capacity region. This claim is somewhat surprising, since the Nash equilibrium is in general inefficient compared to the Pareto optimality. However, their results rely on the fact that both transmitters have complete knowledge of the CSI, and in particular, perfect CSI of all transmitters in the network. As we previously pointed out, this assumption is rarely realistic in practice.

Thus, this power allocation game needs to be reconstructed with some realistic assumptions made about the knowledge level of mobile devices. Under this consideration, it is of great interest to investigate scenarios in which devices have "incomplete information" about their components, for example, a device is aware of its own channel gain, but unaware of the channel gains of other devices. In game theory, a strategic game with incomplete information is called a "Bayesian game." Over the last ten years, Bayesian game-theoretic tools have been used to design distributed resource allocation strategies only in a few contexts, for example, CDMA networks [13, 14], multicarrier interference networks [15]. The primary motivation of this paper is therefore to investigate how Bayesian games can be applied to study the resource allocation problems in the fading MAC. In some sense, this study can help to design a self-organizing femto-cell network where different frequency bands or subcarriers are used for the femto-cell coverage, for example, different femto-cells operate on different frequency bands to avoid interference.

In this paper, we introduce a Bayesian game-theoretic model to design and analyze the resource allocation problem in a fading MAC, where users are assumed to selfishly maximize their ergodic capacity with incomplete information about the fading channel gains. In such a game-theoretic study, the central question is whether a Bayesian equilibrium exists, and if so, whether the network operates efficiently at the equilibrium point. We prove that there exists exactly one Bayesian equilibrium in our game. Furthermore, we study the network sum-rate maximization problem by assuming that all users coordinate to an optimization-based symmetric strategy. This centralized strategy is important when the fading processes for all users are relatively stationary and the global system structure is fixed for a long period of time. This result also serves as an upper bound for the unique Bayesian equilibrium.

The paper is organized in the following form: In Section 2, we introduce the system model and state important assumptions. In Section 3, the K-user MAC is formulated as a static Bayesian game. In Section 4, we characterize the Bayesian equilibrium set. In Section 5, we give a special discussion on the optimal symmetric strategy. Some numerical results are provided to show the efficiency of the Bayesian equilibrium in Section 6. Finally, we close with some concluding remarks in Section 7.

2. System Model and Assumptions

2.1. System Model. We consider the uplink of a single-cell network where K users are simultaneously sending information to one base station. This corresponds to a fading MAC, in which the users are the transmitters and the base station is the receiver. The signal received at the base station can be mathematically expressed as

$$y(t) = \sum_{k=1}^{K} \sqrt{g_k(t)} x_k(t) + z(t), \tag{1}$$

where $x_k(t)$ and $g_k(t)$ are the input signal and fading channel gain of user k, and $z(t)$ is a zero-mean white Gaussian noise with variance σ^2. The input signal $x_k(t)$ can be further written as

$$x_k(t) = \sqrt{p_k(t)} s_k(t), \tag{2}$$

where $p_k(t)$ and $s_k(t)$ are the transmitted power and data of user k at time t.

In this study, we consider the wireless transmission in fast fading environments, that is, the coherence time of the channel is small relative to the delay constraint of the application. When the receiver can perfectly track the channel but the transmitters have no such information, the codewords cannot be chosen as a function of the state of the channel but the decoding can make use of such information. When the fading process is assumed to be stationary and ergodic within the considered interval of signal transmission, the channel capacity in a fast fading channel corresponds to the notion of ergodic capacity, that is,

$$\mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{g_k p_k}{\sigma^2 + \sum_{j=1, j \neq k}^{K} g_j p_j} \right) \right], \tag{3}$$

where $\mathbf{g} = \{g_1, \ldots, g_K\}$ is a vector of channel gain variables. Note that in (3) we assume that the receiver applies a single-user decoding and there is not sophisticated successive decoding to be used. An intuitive understanding of this result can be obtained by viewing capacities in terms of time averages of mutual information [16]. Although the study of multiuser decoding is important, which may involve Stackelberg games, fairness concepts, and generalized Nash games, it is not provided in this study. The interested readers are referred to [17] for this topic.

2.2. Assumption of Finite Channel States. Before introducing our game model, we need to clarify a prior assumption for this section.

Assumption 1. We assume that each user's channel gain g_k is i.i.d. from two discrete values: g_- and g_+ with probability ρ_- and ρ_+, respectively. Without loss of generality, we assume $g_- < g_+$.

On the one hand, our assumption is closely related to the way how feedback information is signalled to the transmitters. In order to get the channel information g_k at the transmitter side, the base station is required to feedback an estimate of g_k to user k *at a given precision*. Since in digital communications, any information is represented by a finite number of bits (e.g., x bits), channels gains are mapped into a set that contains a finite number of states (2^x states).

On the other hand, this is a necessary assumption for analytical tractability, since in principle the functional strategic form of a player can be quite complex with both actions and states being continuous (or infinite). To avoid this problem, in [15] the authors successfully modelled a multicarrier Gaussian interference channel as a Bayesian game with discrete (or finite) actions and continuous states. Inspired from [15], we also model the fading MAC as a Bayesian game under the assumption of continuous actions and discrete states.

3. Game Formulation

We model the K-user fading MAC as a Bayesian game, in which users do not have complete information. In a K-user MAC, to have "complete information" means that, at each time t, the channel gain realizations $g_1(t), \dots, g_K(t)$ are known at all the transmitters, denoted by $\mathrm{Tx}_1, \dots, \mathrm{Tx}_K$. Any other condition corresponds to a situation of incomplete information. In this paper, the "incomplete information" particularly refers to a situation where each Tx_k only knows its own channel gain realization $g_k(t)$, but does not know the channel gains of other transmitters $\mathrm{g}_{-k}(t) = \{g_1(t), \dots, g_{k-1}(t), g_{k+1}(t), \dots, g_K(t)\}$. We will denote by g_k the channel gain variable of user k, whose distribution is assumed to be stationary and ergodic in this section.

In such a communication system, the natural object of each user is to maximize its ergodic capacity subject to an average power constraint, that is,

$$\begin{aligned} \max_{p_k} \quad & \mathbb{E}_{\mathbf{g}}\left[\log\left(1 + \frac{g_k p_k(g_k)}{\sigma^2 + \sum_{j \neq k} g_j p_j(g_j)}\right)\right] \\ \text{s.t.} \quad & \mathbb{E}_{g_k}[p_k(g_k)] \le P_k^{\max} \\ & p_k(g_k) \ge 0, \end{aligned} \tag{4}$$

where $p_k(\cdot)$ and $P_k^{\max}$ are transmit power strategy and average power constraint of user k, respectively. Under the assumption that each user has incomplete information about the channel gains, user k's strategy $p_k(\cdot)$ is defined as a function of its own channel gain g_k, that is, $p_k(g_n)$. Note that (4) implies that user k should know at least the statistics of other users' channels.

For a given set of power strategies $\mathbf{p}_{-k} = \{p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_K\}$, the single-user maximization problem (4) is a convex optimization problem [18]. Via Lagrangian duality, the solution is given by the following equation:

$$\mathbb{E}_{\mathbf{g}_{-k}}\left[\frac{g_k}{\sigma^2 + g_k p_k(g_k) + \sum_{j \neq k} g_j p_j(g_j)}\right] = \lambda_k, \tag{5}$$

where the dual variable λ_k is chosen such that the power constraint in (4) is satisfied with equality. However, the solution of (5) depends on $\mathbf{p}_{-k}(\cdot)$ which user k does not know, and the same holds for all other users. Thus, in order to obtain the optimal power allocation, each user must adjust its power level based on the guess of all other users' strategies. Now, given the following game model, each user is able to adjust its strategy according to the belief it has about the strategy of the other user.

The K-player MAC Bayesian game can be completely characterized as

$$\mathcal{G}_{\mathrm{MAC}} \triangleq \langle \mathcal{K}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{U} \rangle. \tag{6}$$

(i) Player set: $\mathcal{K} = \{1, \dots, K\}$.

(ii) Type set: $\mathcal{T} = \mathcal{T}_1 \times \cdots \times \mathcal{T}_K$ ("$\times$" stands for the Cartesian product) where $\mathcal{T}_k = \{g_-, g_+\}$. A player's type is defined as its channel gain, that is, $g_k \in \mathcal{T}_k$.

(iii) Action set: $\mathcal{P} = \mathcal{P}_1 \times \cdots \times \mathcal{P}_K$ where $\mathcal{P}_k = [0, P_k^{\max}]$. A player's action is defined as its transmit power, that is, $p_k \in \mathcal{P}_k$.

(iv) Probability set: $\mathcal{Q} = \mathcal{Q}_1 \times \cdots \times \mathcal{Q}_K$ where $\mathcal{Q}_k = \{\rho_-, \rho_+\}$, we have $\rho_+ = \Pr(g_k = g_+)$ and $\rho_- = \Pr(g_k = g_-)$.

(v) Payoff function set: $\mathcal{U} = \{u_1, \dots, u_K\}$ where u_k is chosen as player k's achievable rate

$$u_k(p_1, \dots, p_K) = \log\left(1 + \frac{g_k p_k(g_k)}{\sigma^2 + \sum_{j=1, j \neq k}^{K} g_j p_j(g_j)}\right). \tag{7}$$

In games of incomplete information, a player's type represents any kind of private information that is relevant to its decision making. In our context, the fading channel gain g_k is naturally considered as the type of user k, since its decision (in terms of power) can only rely on g_k. Note that this is a *continuous game* (a continuous game extends the notion of a discrete game (where players choose from a finite set of pure strategies), it allows players to choose a strategy from a continuous pure strategy set) with discrete states, since each player's action p_k can take any value satisfying the constraint $p_k \in [0, P_k^{\max}]$ and the channel state g_k is finite $g_k \in \{g_-, g_+\}$.

4. Bayesian Equilibrium

4.1. Definition of Bayesian Equilibrium. What we can expect from the outcome of a Bayesian game if every selfish and rational (rational player means a player chooses the best response given its information) participant starts to play the game? Generally speaking, the process of such players' behaviors usually results in a Bayesian equilibrium, which represents a common solution concept for Bayesian games. In many cases, it represents a "stable" result of learning and evolution of all participants. Therefore, it is important to characterize such an equilibrium point, since it concerns the performance prediction of a distributed system.

Now, let $\{\hat{p}_k(\cdot), \mathbf{p}_{-k}(\cdot)\}$ denote the strategy profile where all players play $p(\cdot)$ except player k who plays $\hat{p}_k(\cdot)$, we can then describe player k's payoff as

$$u_k(\hat{p}_k, \mathbf{p}_{-k}) = u_k(p_1, \dots, p_{k-1}, \hat{p}_k, p_{k+1}, \dots, p_K). \quad (8)$$

Definition 2 (Bayesian equilibrium). The strategy profile $p^\star(\cdot) = \{p_k^\star(\cdot)\}_{k\in\mathcal{K}}$ is a (pure strategy) Bayesian equilibrium, if for all $k \in \mathcal{K}$, and for all $p_k(\cdot) \in \mathcal{P}_k$ and $\mathbf{p}_{-k}(\cdot) \in \mathcal{P}_{-k}$

$$\overline{u}_k\left(p_k^\star, \mathbf{p}_{-k}^\star\right) \geq \overline{u}_k\left(p_k, \mathbf{p}_{-k}^\star\right), \quad (9)$$

where we define $\overline{u}_n \triangleq \mathbb{E}_{\mathbf{g}}[u_k]$.

From this definition, it is clear that at the Bayesian equilibrium no player can benefit from changing its strategy while the other players keep theirs unchanged. Note that in a strategic-form game with complete information each player chooses a concrete action, whereas in a Bayesian game each player k faces the problem of choosing a set or collection of actions (power strategy $p_k(\cdot)$), one for each type (channel gain g_k) it may encounter. It is also worth to mention that the action set of each player is independent of the type set, that is, the actions available to user k are the same for all types.

4.2. Characterization of the Bayesian Equilibrium Set. It is well known that, in general, an equilibrium point does not necessarily exist [6]. Therefore, our primary interest in this paper is to investigate the *existence* and *uniqueness* of a Bayesian equilibrium in $\mathcal{G}_{\text{MAC}}$. We now state our main result.

Theorem 3. *There exists a unique Bayesian equilibrium in the K-user* MAC *game $\mathcal{G}_{\text{MAC}}$.*

Proof. It is easy to prove the existence part, since the strategy space p_k is convex, compact, and nonempty for each k; the payoff function u_k is continuous in both p_k and $\mathbf{p}_{-k}$; u_k is concave in p_k for any $\mathbf{p}_{-k}$ [6].

In order to prove the uniqueness part, we should rely on a sufficient condition given in [19]: a non-cooperative game has a unique equilibrium, if the nonnegative weighted sum of the payoff functions is *diagonally strictly concave*. We firstly give the definition.

Definition 4 (diagonally strictly concave). A weighted nonnegative sum function $f(\mathbf{x}, \mathbf{r}) = \sum_{i=1}^n r_i\varphi_i(\mathbf{x})$ is called diagonally strictly concave for any vector $\mathbf{x} \in \mathbb{R}^{n\times 1}$ and fixed vector $\mathbf{r} \in \mathbb{R}_{++}^{n\times 1}$, if for any two different vectors $\mathbf{x}^0, \mathbf{x}^1$, we have

$$\Omega(\mathbf{x}^0, \mathbf{x}^1, \mathbf{r}) \triangleq (\mathbf{x}^1 - \mathbf{x}^0)^{\mathrm{T}}\delta(\mathbf{x}^0, \mathbf{r}) + (\mathbf{x}^0 - \mathbf{x}^1)^{\mathrm{T}}\delta(\mathbf{x}^1, \mathbf{r}) > 0, \quad (10)$$

where $\delta(\mathbf{x}, \mathbf{r})$ is called pseudogradient of $f(\mathbf{x}, \mathbf{r})$, defined as

$$\delta(\mathbf{x}, \mathbf{r}) \triangleq \begin{bmatrix} r_1 \dfrac{\partial\varphi_1}{\partial x_1} \\ \vdots \\ r_n \dfrac{\partial\varphi_n}{\partial x_n} \end{bmatrix}. \quad (11)$$

We start with the following lemma.

Lemma 5. *The weighted nonnegative sum of the average payoffs $\overline{u}_k$ in $\mathcal{G}_{\text{MAC}}$ is diagonally strictly concave for $\mathbf{r} = c^+\mathbf{1}$, where c^+ is a positive scalar, $\mathbf{1}$ is a vector whose every entry is* 1.

Proof. Write the weighted nonnegative sum of the average payoffs as

$$f^u(\mathbf{p}, \mathbf{r}) \triangleq \sum_{k=1}^K r_k\overline{u}_k(\mathbf{p}), \quad (12)$$

where $\mathbf{p} = [p_1 \cdots p_K]^{\mathrm{T}}$ is the transmit power vector and $\mathbf{r} = [r_1 \cdots r_K]^{\mathrm{T}}$ is a nonnegative vector assigning weights $r_1, \dots, r_K$ to the average payoffs $\overline{u}_1, \dots, \overline{u}_K$, respectively. Similar to (11), we let $\delta^u(\mathbf{p}, \mathbf{r}) \triangleq [r_1(\partial\overline{u}_1/\partial p_1)\dots r_K(\partial\overline{u}_K/\partial p_K)]^{\mathrm{T}}$ be the pseudogradient of $f^u(\mathbf{p}, \mathbf{r})$. Now, we define

$$p_k \triangleq p_k(g_-) \quad \forall k, \quad (13)$$

the transmit power of player k when its channel gain is g_-. Since we have shown from the Lagrangian that, at the equilibrium, the power constraint is satisfied with equality, that is, $\mathbb{E}_{g_k}[p_k(g_k)] = P_k^{\max}$, we can write $P_k^{\max} - \rho_- p_k(g_-) = \rho_+ p_k(g_+)$ for all k, as the transmit power when its channel gain is g_+. Therefore, it is easy to find that the average payoff $\overline{u}_k$ can be actually transformed into a weighted sum-log function as follows:

$$\overline{u}_k(p_k) = \sum_i \omega_i \log\left[1 + \frac{\alpha_k^i + \beta_k^i p_k}{\sigma^2 + \sum_{j\neq k}\left(\alpha_j^i + \beta_j^i p_j\right)}\right], \quad (14)$$

where i represents the index for different jointly probability events, ω_i represents the corresponding probability for event i that are related to the probabilities $\{\rho_-, \rho_+\}$, and α_k^i and β_n^i represent some positive and nonzero real numbers that are related to the channel gains $\{g_-, g_+\}$. Note that the following conditions hold for all i, k:

$$\omega_i > 0, \quad \alpha_k^i + \beta_k^i p_k \geq 0, \quad \alpha_k^i > 0, \quad \beta_k^i \neq 0, \quad \sigma^2 > 0. \quad (15)$$

Now, we can write the pseudogradient δ^u as

$$\begin{aligned} \delta^u(\mathbf{p}, \mathbf{r}) &= \begin{bmatrix} c^+\dfrac{\partial\overline{u}_1}{\partial p_1} \\ \vdots \\ c^+\dfrac{\partial\overline{u}_K}{\partial p_K} \end{bmatrix} \\ &= \begin{bmatrix} c^+\sum_i \omega_i\beta_1^i\phi_i^{-1}(\mathbf{p}) \\ \vdots \\ c^+\sum_i \omega_i\beta_K^i\phi_i^{-1}(\mathbf{p}) \end{bmatrix} \\ &= c^+\sum_i \begin{bmatrix} \omega_i\beta_1^i\phi_i^{-1}(\mathbf{p}) \\ \vdots \\ \omega_i\beta_K^i\phi_i^{-1}(\mathbf{p}) \end{bmatrix}, \end{aligned} \quad (16)$$

where the function $\phi_i(\mathbf{x})$ is defined as

$$\phi_i(\mathbf{x}) \triangleq \sigma^2 + \sum_{k=1}^{K} \left(\alpha_k^i + \beta_k^i x_k\right). \quad (17)$$

To check the diagonally strictly concave condition (10), we let $\mathbf{p}^0, \mathbf{p}^1$ be two different vectors satisfying the power constraint, and define

$$\begin{aligned}
\Omega^u(\mathbf{p}^0, \mathbf{p}^1, \mathbf{r}) &\triangleq (\mathbf{p}^1 - \mathbf{p}^0)^{\mathrm{T}} \delta^u(\mathbf{p}^0, \mathbf{r}) + (\mathbf{p}^0 - \mathbf{p}^1)^{\mathrm{T}} \delta^u(\mathbf{p}^1, \mathbf{r}) \\
&= (\mathbf{p}^1 - \mathbf{p}^0)^{\mathrm{T}} [\delta^u(\mathbf{p}^0, \mathbf{r}) - \delta^u(\mathbf{p}^1, \mathbf{r})] \\
&= [\Delta p_1 \ \cdots \ \Delta p_K] \\
&\quad \times \begin{bmatrix} c^+ \sum_i \omega_i \beta_1^i (\phi_i^{-1}(\mathbf{p}^0) - \phi_i^{-1}(\mathbf{p}^1)) \\ \vdots \\ c^+ \sum_i \omega_i \beta_K^i (\phi_i^{-1}(\mathbf{p}^0) - \phi_i^{-1}(\mathbf{p}^1)) \end{bmatrix} \\
&= c^+ \sum_i \omega_i [\phi_i^{-1}(\mathbf{p}^0) - \phi_i^{-1}(\mathbf{p}^1)] \zeta_i \\
&= c^+ \sum_i \omega_i \phi_i^{-1}(\mathbf{p}^0) \phi_i^{-1}(\mathbf{p}^1) \zeta_i^2,
\end{aligned} \quad (18)$$

where Δp_k and ζ_i are defined as

$$\begin{aligned}
\Delta p_k &\triangleq p_k^1 - p_k^0, \\
\zeta_i &\triangleq \sum_{k=1}^{K} \beta_k^i \Delta p_k,
\end{aligned} \quad (19)$$

Since $\mathbf{p}^0$, $\mathbf{p}^1$ are assumed to be two different vectors, we must have $\Delta\mathbf{p} = [\Delta p_1 \cdots \Delta p_K]^{\mathrm{T}} \neq \mathbf{0}$. Now, we can draw a conclusion from the equation above: $\Omega^u(\mathbf{p}^0, \mathbf{p}^1, \mathbf{r}) > 0$. This is because: (1) the first part $\omega_i \phi_i^{-1}(\mathbf{p}^0) \phi_i^{-1}(\mathbf{p}^1) > 0$ for all i, since $\omega_i > 0, \sigma^2 > 0$ and $\alpha_k^i + \beta_k^i p_k \geq 0$ for all i, k; (2) the second part $\zeta_i^2 \geq 0$ for all i, and there exists at least one nonzero term ζ_i^2, due to $\Delta\mathbf{p} \neq 0$ and $r_k \neq 0$, $\beta_k^i \neq 0$ for all i, k. Therefore, the summation of all the products of the first and the second terms must be positive. From Definition 4, the sum-payoff function $f^u(\mathbf{p}, \mathbf{r})$ satisfies the condition of diagonally strictly concave. This completes the proof of this lemma. □

Since our sum-payoff function $f^u(\mathbf{p}, \mathbf{r})$ given in (12) is diagonally strictly concave, the uniqueness of Bayesian equilibrium in our game $\mathcal{G}_{\mathrm{MAC}}$ follows directly from [19, Theorem 2]. □

5. Optimal Symmetric Strategies

The Bayesian game-theoretic approach provides us a better understanding of the wireless resource competition existing in the fading MAC when every mobile device acts as a selfish and rational decision maker (this means a device always chooses the best response given its information). The advantage of this model is that it mathematically captures the behavior of selfish wireless entities in strategic situations, which can automatically lead to the convergence of system performance. The introduced Bayesian game-theoretic framework fits very well the concept of self-organizing networks, where the intelligence and decision making is distributed. Such a scheme has apparent benefits in terms of operational complexity and feedback load.

However, from the global system performance perspective, it is usually inefficient to give complete "freedom" to mobile devices and let them take decisions without any policy control over the network. It is very interesting to note that a similar situation happens in the market economy, where consumers can be modeled as players to complete for the market resources. In the famous literature *The Wealth of Nations*, Adam Smith (a Scottish moral philosopher, pioneer of political economy, and father of modern economics) expounded how rational self-interest and competition can lead to economic prosperity and well-being through macroeconomic adjustments. For example, all states today have some form of macroeconomic control over the market that removes the free and unrestricted direction of resources from consumers and prices such as tariffs and corporate subsidies.

In particular, wireless service providers would like to design an appropriate policy to efficiently manage the system resource so that the global network performance can be optimized or enhanced to a certain theoretical limit, for example, Shannon capacity or capacity region [20]. Apparently, a centralized scheduler with comprehensive knowledge of the network status can globally optimize the resource utility. However, this approach usually involves sophisticated optimization techniques and a feedback load that grows with the number of wireless devices in the network. Thus, the optimization-based centralized decision has to be frequently updated as long as the wireless environment varies, or the system structure changes, for example, a user joins or exits the network.

In this section, we consider that the channel statistics (fading processes) for all wireless devices are jointly stationary for a relatively long period of signal transmission, and the global system structure remains unchanged. In addition, we neglect the problem of computational complexity at the scheduler and the impact of feedback load to the useful data transmission rate. In this case, the network service provider would strictly prefer to use a centralized approach, that is, a scheduler assigns some globally optimal strategies to the wireless devices, guiding them how to react under all kinds of different situations. Based on the Bayesian game settings, we provide a special discussion on the optimal symmetric strategy design. Note that this result can be treated as a theoretical upperbound for the performance measurement of Bayesian equilibrium.

We now introduce a necessary assumption.

Assumption 6. Mobile devices are designed to use the same power strategies, that is, they send the same power if their observations on the channel states are symmetric. In addition, we assume that the mobile devices have the same average power constraint, that is, $P_1^{\max} = \cdots = P_K^{\max} \triangleq P^{\max}$.

5.1. Two Channel States. For simplicity of our presentation, We first consider the scenario of two users with two channel states. In fact, the analysis of multiuser MAC can be extended in a similar way. According to Assumption 6, we define

$$p_- \triangleq p_1(g_-) = p_2(g_-),$$
$$p_+ \triangleq p_1(g_+) = p_2(g_+), \tag{20}$$

and we have $\rho_- p_- + \rho_+ p_+ = P^{\max}$. Write user 1's average payoff as (Without loss of generality, we consider user 1 in the following context, since the problem is symmetric for user 2)

$$\begin{aligned}
\overline{u}_1 &= \mathbb{E}_{g_1,g_2}\left[\log_2\left(1+\frac{g_1 p_1(g_1)}{\sigma^2 + g_2 p_2(g_2)}\right)\right] \\
&= \rho_-^2 \log_2\left(1+\frac{g_- p_-}{\sigma^2 + g_- p_-}\right) \\
&\quad + \rho_-\rho_+ \log_2\left(1+\frac{g_- p_-}{\sigma^2 + g_+(P^{\max} - \rho_- p_-)/\rho_+}\right) \\
&\quad + \rho_-\rho_+ \log_2\left(1+\frac{g_+(P^{\max} - \rho_- p_-)/\rho_+}{\sigma^2 + g_- p_-}\right) \\
&\quad + \rho_+^2 \log_2\left(1+\frac{g_+(P^{\max} - \rho_- p_-)/\rho_+}{\sigma^2 + g_+(P^{\max} - \rho_- p_-)/\rho_+}\right).
\end{aligned} \tag{21}$$

Now, $\overline{u}_1$ is transformed into a function of p_-, write it as $\overline{u}_1(p_-)$. To maximize the average achievable rate, user 1 needs to solve the following optimization problem, as mentioned in (4)

$$\begin{aligned}
\max_{p_-} \quad & \overline{u}_1(p_-) \\
\text{s.t.} \quad & 0 \le p_- \le \frac{P^{\max}}{\rho_-}.
\end{aligned} \tag{22}$$

Under Assumption 6, it can be shown that (due to the symmetric property) this single-user maximization problem is equivalent to the multiuser sum average rate maximization problem, that is, $\max(\overline{u}_1 + \overline{u}_2)$, which is our object in this section.

But unfortunately, $\overline{u}_1$ may not be a convex function [18], so the single-user problem may not be a convex optimization problem. It can be further verified that $\overline{u}_1$ is convex under some special conditions, depending on all the parameters $g_-, g_+, \rho_-, \rho_+, P^{\max}$, and σ^2. Here, we will not discuss all the convex cases, but only focus on the high SNR regime (meaning that the noise can be omitted compared to the signal strength). In this case, we have

$$\begin{aligned}
\lim_{\sigma^2 \to 0} u_1 = \rho_-\rho_+ \Bigg[& \log_2\left(1+\frac{g_- p_-}{g_+(P^{\max} - \rho_- p_-)/\rho_+}\right) \\
& + \log_2\left(1+\frac{g_+(P^{\max} - \rho_- p_-)/\rho_+}{g_- p_-}\right)\Bigg] \\
& + \rho_-^2 + \rho_+^2.
\end{aligned} \tag{23}$$

This function is strict convex. To be more precise, it is decreasing on $[0, g_+ P^{\max}/(g_-\rho_+ + g_+\rho_-))$ and increasing on $(g_+ P^{\max}/(g_-\rho_+ + g_+\rho_-), P^{\max}/\rho_-]$, and the solution is given by

$$\{p_-^\star, p_+^\star\} = \begin{cases} \left\{0, \dfrac{P^{\max}}{\rho_+}\right\}, & \dfrac{g_+}{\rho_+} \ge \dfrac{g_-}{\rho_-}, \\[2ex] \left\{\dfrac{P^{\max}}{\rho_-}, 0\right\}, & \dfrac{g_+}{\rho_+} < \dfrac{g_-}{\rho_-}. \end{cases} \tag{24}$$

Note that in this setting the choice of the optimal symmetric strategy is to concentrate the full available power on a single channel state. The selection of the channel state on which to transmit depends not only on the channel conditions but also on the probability of the channel states. This result implies that, in the high SNR regime, the optimal symmetric power strategy is to transmit information in an "opportunistic" way. For a better understanding of the "opportunistic" transmission, the interested reads are referred to [2].

5.2. Multiple Channel States. In this subsection, we discuss the extension to arbitrary L ($L \ge 2$) channel states. Note that the result of this subsection can also be applied to the case of two channel states.

Assumption 7. Each user's channel gain g_k has L positive states, which are $a_1, \ldots, a_L$ with probability $\rho_1, \ldots, \rho_L$, respectively (Without loss of generality,$a_1 < \cdots < a_L$), and we have $\sum_{\ell=1}^{L} \rho_\ell = 1$.

Based on Assumption 6, we define $p_\ell \triangleq p_1(a_\ell) = p_2(a_\ell), \ell = 1, \ldots, L$, as the transmit power when a user's channel gain is a_ℓ. As previously mentioned, our object in this part is to maximize the sum ergodic capacity of the system, that is, $\max \sum_k \overline{u}_k$. Under the symmetric assumption, this sum-ergodic-capacity maximization problem is equivalent to the following single-user maximization problem

$$\begin{aligned}
\max_{\mathbf{p}} \quad & \sum_i \sum_j \rho_i \rho_j \log_2\left(1+\frac{g_i p_i}{\sigma^2 + g_j p_j}\right) \\
\text{s.t.} \quad & \sum_i \rho_i p_i \le P^{\max} \\
& p_i \ge 0, \quad i = 1, \ldots, L,
\end{aligned} \tag{25}$$

where $\mathbf{p}$ is now defined as $\mathbf{p} = \{p_1, \ldots, p_L\}$. This optimization problem is difficult, since the objective function is again nonconvex in $\mathbf{p}$. However, we can consider a relaxation of the optimization by introducing a lower bound [21]

$$\alpha \log z + \beta \le \log(1+z), \tag{26}$$

where α and β are chosen specified as

$$\begin{aligned}
\alpha &= \frac{z_0}{1+z_0}, \\
\beta &= \log(1+z_0) - \frac{z_0}{1+z_0} \log z_0,
\end{aligned} \tag{27}$$

we say that the lower bound (26) is tight with equality at a chosen value z_0.

Let us consider the lower bound (denoted as ξ) by using the relaxation (26) to the objective function in (25)

$$\xi(\mathbf{p}) \triangleq \sum_i \sum_j \rho_i \rho_j \left[\alpha_{i,j} \log_2 \left(\frac{g_i p_i}{\sigma^2 + g_j p_j} \right) + \beta_{i,j} \right] \tag{28}$$

which is still nonconvex, and so it is not concave in $\mathbf{p}$. However, with a logarithmic change of the following variables and constants: $\tilde{p}_i = \log_2 p_i$, $\tilde{\mathbf{p}}_i = \log_2 \mathbf{p}_i$ and $\tilde{g}_i = \log_2 g_i$, we can turn the geometric programming [18] associated with the objective function (28) into the following problem:

$$\begin{aligned} \max_{\tilde{\mathbf{p}}} \quad & \xi(\tilde{\mathbf{p}}) \\ \text{s.t.} \quad & \sum_i \rho_i 2^{\tilde{p}_i} \le P^{\max}, \end{aligned} \tag{29}$$

where $\xi(\tilde{\mathbf{p}})$ is defined as

$$\begin{aligned} \xi(\tilde{\mathbf{p}}) = & \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} (\tilde{g}_i + \tilde{p}_i) \\ & - \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} \log_2 \left(\sigma^2 + 2^{(\tilde{g}_j + \tilde{p}_j)} \right) \\ & + \sum_i \sum_j \rho_i \rho_j \beta_{i,j}. \end{aligned} \tag{30}$$

Now, it is easy to verify that the lower bound ξ is concave in the transformed set $\tilde{\mathbf{p}}$, since the log-sum-exp function is convex. The constraints of the optimization problem are such that Slater's condition is satisfied [18]. So, the Karush-Kuhn-Tucker (KKT) condition of the optimization is sufficient and necessary for the optimality. Given the following Lagrangian dual function (denoted by $\mathcal{L}$):

$$\begin{aligned} \mathcal{L}(\tilde{\mathbf{p}}, \nu) = & \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} (\tilde{a}_i + \tilde{p}_i) \\ & - \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} \log_2 \left(\sigma^2 + 2^{(\tilde{a}_j + \tilde{p}_j)} \right) \\ & + \sum_i \sum_j \rho_i \rho_j \beta_{i,j} - \nu \left(\sum_i 2^{(\tilde{a}_i + \tilde{p}_i)} - P^{\max} \right), \end{aligned} \tag{31}$$

the KKT conditions are

$$\begin{aligned} & \rho_\ell \sum_j \rho_j \alpha_{\ell,j} - \rho_\ell \left(\frac{2^{(\tilde{a}_\ell + \tilde{p}_\ell)}}{\sigma^2 + 2^{(\tilde{a}_\ell + \tilde{p}_\ell)}} \right) \\ & \quad \times \sum_i \rho_i \alpha_{i,\ell} - (\nu \ln 2) 2^{(\tilde{a}_\ell + \tilde{p}_\ell)} = 0, \quad \forall \ell, \end{aligned} \tag{32}$$

where $\tilde{a}_\ell = \log_2 a_\ell$, and $\nu \ge 0$ is a dual variable associated with the power constraint in (29).

Define $x_\ell \triangleq 2^{(\tilde{a}_\ell + \tilde{p}_\ell)}, \ell = 1, \ldots, L$, the equivalent KKT conditions can be simply written as a quadratic equation

$$A_\ell x_\ell^2 + B_\ell x_\ell + C_\ell = 0, \quad \forall \ell, \tag{33}$$

where the parameters A_ℓ, B_ℓ, C_ℓ are expressed as

$$\begin{aligned} A_\ell &= \nu \ln 2, \quad \forall \ell \\ B_\ell &= \rho_\ell \sum_i \rho_i (\alpha_{i,\ell} - \alpha_{\ell,i}) + \sigma^2 \nu \ln 2, \quad \forall \ell \\ C_\ell &= -\rho_\ell \sigma^2 \sum_i \rho_i \alpha_{\ell,i}, \quad \forall \ell. \end{aligned} \tag{34}$$

Note that A_ℓ and B_ℓ are functions of ν, we can write them as $A_\ell(\nu)$ and $B_\ell(\nu)$. Since $x_\ell \ge 0$, the solution to the KKT conditions can only be one of the roots to the quadratic equation, that is,

$$p_\ell^\star = \frac{-B_\ell(\nu) + \sqrt{B_\ell^2(\nu) - 4A_\ell(\nu)C_\ell}}{2a_\ell A_\ell(\nu)}, \quad \forall \ell, \tag{35}$$

where ν is chosen such that $\sum_\ell \rho_\ell p_\ell^\star = P^{\max}$. Thus, for some fixed value of α, β, we can directly apply (35) to maximize the lower bound ξ (28). Then, it is natural to improve the bound periodically. Based on the discussion above, we propose the following algorithm, namely Lower Bound Tightening (LBT) algorithm

The algorithm convergence can be easily proved, since the objective is monotonically increasing at each iteration. However, the global optimum is not always guaranteed, due to the nonconvex property.

6. Numerical Results

In this section, numerical results are presented to validate our theoretical claims. For Figures 1 and 2, the network parameters are chosen as $\rho_- = \rho_+ = 0.5$, $P^{\max} = 1$ and $\sigma^2 = 0.1$.

First, we show the existence and uniqueness of Bayesian equilibrium in the scenario of two-user fading MAC. In Figure 1(a), we assume the channel gains are $g_- = 1$, $g_+ = 3$; in Figure 1(b), we assume $g_- = 1, g_+ = 10$. On both X and Y axis, p_1 and p_2 represent the power allocated by user 1 and user 2 when the channel gain is g_-. The curves $r_1(p_2)$ and $r_2(p_1)$ represent the best-response functions of user 1 and user 2, respectively. As expected, the Bayesian equilibrium is unique in both cases, that is, (0.6,0.6) and (0.5,0.5).

Second, we investigate the efficiency of Bayesian equilibrium from the viewpoint of global average network performance. The X axis, SNR is defined as the ratio between the power constraint $P^{\max}$ and the noise variance σ^2. In Figure 2(a), again, we assume $g_- = 1$, $g_+ = 3$; in Figure 2(b), we assume $g_- = 1$, $g_+ = 10$. The curve "Pareto" represents the Nash equilibrium in the waterfilling game, in which users have complete information. This gives the upper bound for our Bayesian equilibrium, since it is also the Pareto optimal solution [7]. The curve "Uniform"

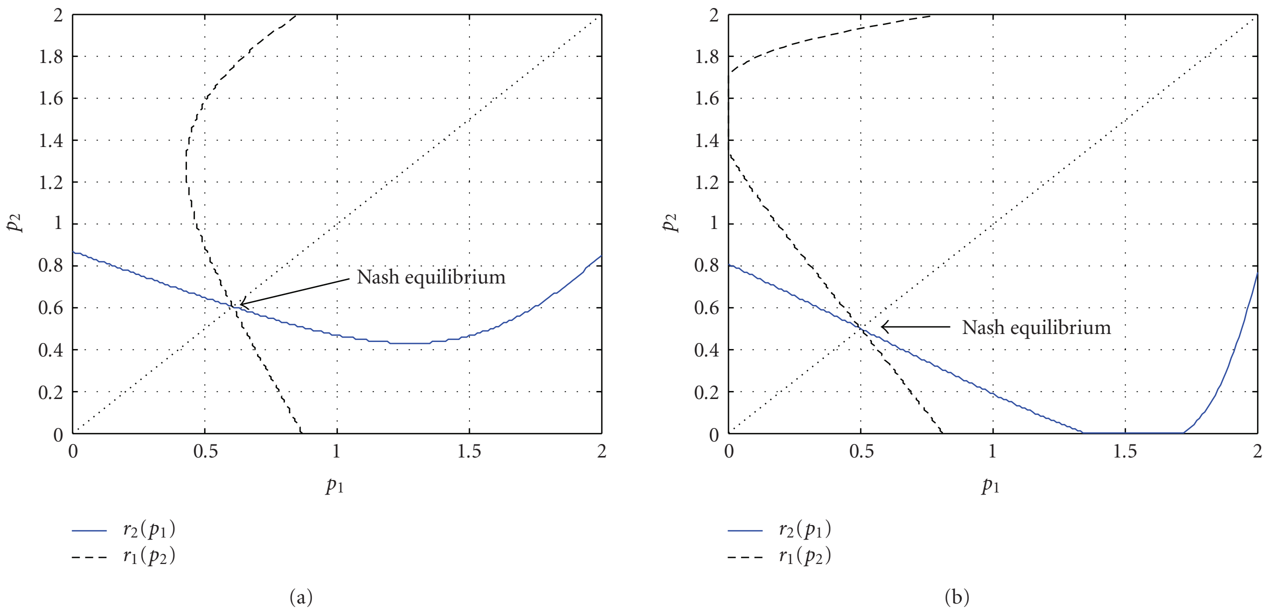

Figure 1: The uniqueness of Bayesian equilibrium. (a) $g_- = 1$, $g_+ = 3$, (b) $g_- = 1$, $g_+ = 10$.

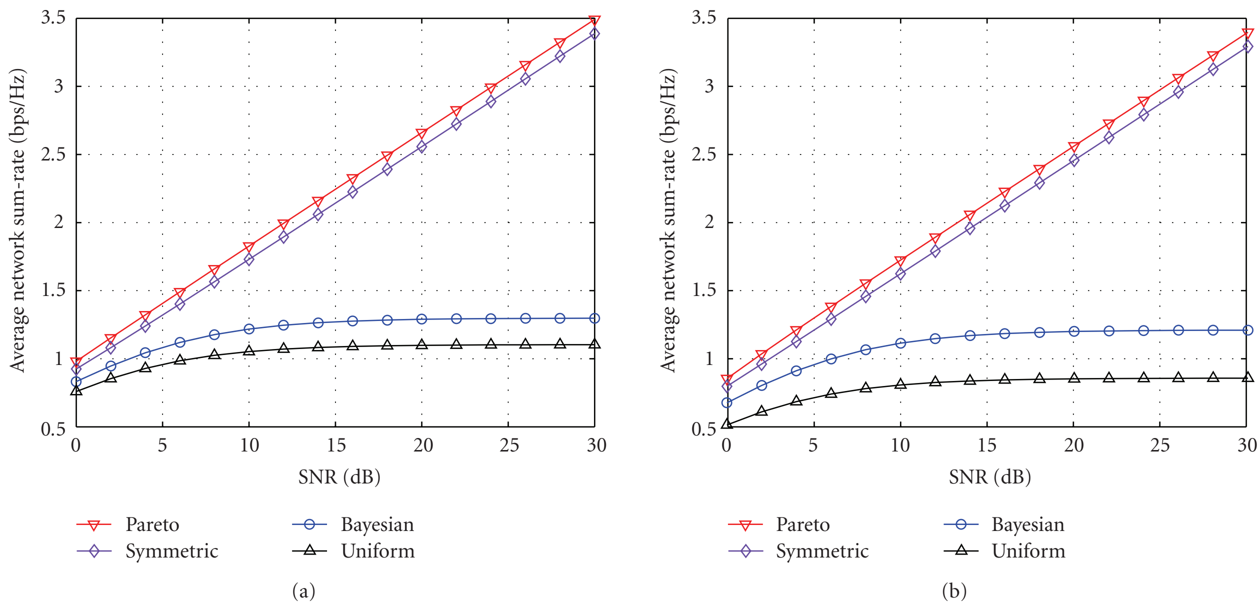

Figure 2: Average network sum-rate. (a) $g_- = 1$, $g_+ = 3$, (b) $g_- = 1$, $g_+ = 10$.

represents the time-domain uniform power allocation. Since this is the strategy when users have no information about the channel gains, it corresponds obviously to a lower bound. The curve "Symmetric" represents the optimal symmetric strategy presented in Section 5. This can be treated as a weaker upper bound (inferior to the Pareto optimality) for the Bayesian equilibrium. From the slopes of these curves, we can clearly observe the inefficiency of the Bayesian equilibrium, especially in the high SNR regime. This can be explained as follows: in our game $\mathcal{G}_{\text{MAC}}$, users with incomplete information improve the global network performance (comparing to the scenario in which the users have no information), however, it does not improve the performance slope.

Finally, we show the convergence behavior of the lower bound tightening (LBT) algorithm. In Figure 3, we choose the parameters as $L = 3$, $g_1 = 1$, $g_2 = 2$, $g_3 = 3$, and $\rho_1 = \rho_2 = \rho_3 = 1/3$. The sum capacity versus the SNR are plotted for five iterations. The upper bound is achieved by exhaustive search. As expected, one can easily observe the convergence behavior. In the low SNR regime, we can find that the algorithm converges to the local instead of the global maximum. However, we also find that the performance of the local optimum is improved while the SNR is increasing.

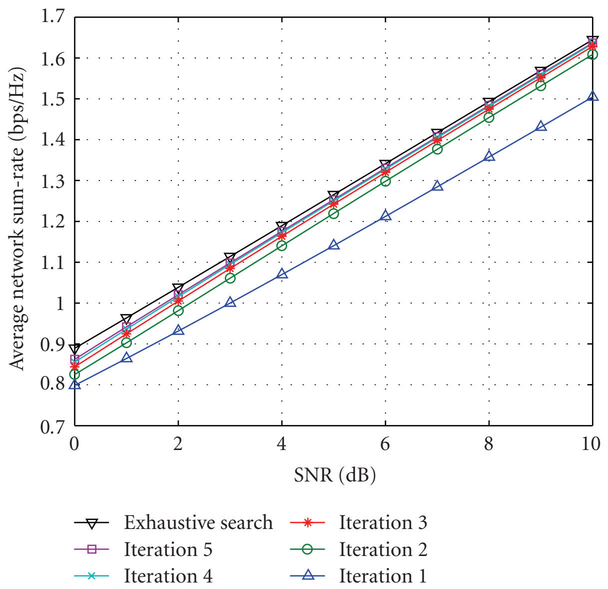

FIGURE 3: The convergence of the lower bound tightening (LBT) algorithm.

Initialize $t = 0$; $\nu = 0$; $\alpha_{i,j}^{(t)} = 1$, for $i = 1, \ldots, L$, $j = 1, \ldots, L$.
repeat
  repeat
    $\nu = \nu + \Delta\nu$
    for $i = 1$ to L **do**
      update A_i, B_i, C_i using (34)
      $p_i^{\star} = (-B_i + \sqrt{B_i^2 - 4A_iC_i})/2a_iA_i$
    end for
  until $\sum_i \rho_i p_i^{\star} = P^{\max}$
  for $i = 1$ to L and $j = 1$ to L **do**
    $z_{i,j}^{(t)} = a_i p_i^{\star}/(\sigma^2 + a_j p_j^{\star})$; $\alpha_{i,j}^{(t+1)} = z_{i,j}^{(t)}/(1 + z_{i,j}^{(t)})$
  end for
  $t = t + 1$
until converge

ALGORITHM 1: Lower Bound Tightening (LBT).

7. Conclusion

We presented a Bayesian game-theoretic framework for distributed resource allocation in fading MAC, where users are assumed to have only information about their own channel gains. By introducing the assumption of finite channel states, we successfully found a analytical way to characterize the Bayesian equilibrium set. First, we proved the existence and uniqueness. Second, the inefficiency was shown from numerical results. Furthermore, we analyzed the optimal symmetric power strategy based on the practical concerns of resource allocation design. Future extension is considered to improve the efficiency of Bayesian equilibrium through pricing or cooperative game-theoretic approaches.

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